

Quantum dynamics of an optomechanical system in the presence of Kerr-down conversion nonlinearity

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(Dated: March 6, 2013)

We study theoretically nonlinear effects arising from the presence of a Kerr-down conversion nonlinear crystal inside an optomechanical cavity. For this system we investigate the influences of the two nonlinearities, i.e., the Kerr nonlinearity and the parametric gain, on the dynamics of the oscillating mirror, the intensity and the squeezing spectra of the transmitted field, and the steady-state mirror-field entanglement. We show that in comparison with a bare optomechanical cavity, the combination of the cavity energy shift due to the Kerr nonlinearity and increase in the intracavity photon number due to the gain medium can increase the normal mode splitting in the displacement spectrum of the oscillating mirror and reduce its effective temperature. Our work demonstrates that both the Kerr nonlinearity and down conversion process increase the degree of squeezing in the transmitted field. Moreover, we find that in the system under consideration the degree of entanglement between the mechanical and optical modes decreases considerably because of the intracavity photon number reduction in the presence of the Kerr medium.

PACS numbers: 37.30.+i, 03.67.Bg, 42.50.Wk, 42.50.Pq

I. INTRODUCTION

In recent years, there has been an increasing interest in cavity optomechanical systems for a wide range of both experimental and theoretical investigations[1–8]. The importance of optomechanical systems is due to their potential applications in different topics of physics. They are promising candidates for studying quantum effects in the mesoscopic and macroscopic scales[9], detection and interferometry of gravitational waves [10–12] and measurement of small displacements[13]. In a cavity optomechanical system the radiation pressure exerted by the electromagnetic field induces a coupling between the intensity of the cavity field and the mechanical motion of a movable mirror. It is pointed out that the necessary condition for observing quantum phenomena for the oscillating mirror, as a mesoscopic or macroscopic object, is the preparation of the mechanical oscillator at low phonon occupancy. Although the ground state cooling of the mirror has not yet been achieved experimentally, it has been shown theoretically[3, 14, 15] that the ground state cooling of the mirror is possible in the resolved sideband regime where the cavity bandwidth is less than mechanical oscillation frequency of the mirror. Another quantum phenomenon expected to be observed in optomechanical systems is the mixing between the fluctuations of the cavity field around the steady state and the mechanical mode which leads to the normal mode splitting (NMS) in the displacement spectrum of the mirror[16]. It is well known from cavity QED that NMS is an absolute ev-

idence of strong coupling between the subsystems with energy exchange taking place on a time scale faster than the decoherence of each mode. The appearance of NMS in optomechanical systems is known as a direct consequence of ground state cooling of the mirror[16].

One of the most important characteristics of optomechanical systems is their nonlinear optical properties. The strong interaction between the cavity field and the mechanical oscillations makes the cavity behave like a nonlinear medium since the length of the cavity depends upon the intensity of the field in analogous way to the optical length of a nonlinear material[8]. It has been shown[17] that within the Born-Openheimer approximation the back action of the oscillating mirror in one edge of an optical Fabry-Perot cavity, driven by a nearly-resonant laser field, induces an intracavity Kerr-like nonlinearity. Besides this intrinsic nonlinearity, the presence of an optical parametric amplifier (OPA) [18] or the optical Kerr medium [19] inside the cavity has opened up a new domain for combining nonlinear optics and optomechanics towards the enhancement of quantum effects. It has been demonstrated[18] that the presence of an OPA in the cavity causes a strong coupling between the oscillating mirror and the cavity mode resulting from increasing the intracavity photon number. Thus the enhancement of radiation pressure-induced coupling not only considerably improves the cooling of the mechanical oscillator but also makes the observation of the NMS of the movable mirror and the output field mode accessible[18]. On the other hand, when the optomechanical cavity contains an optical Kerr medium with $\chi^{(3)}$ nonlinearity, due to the photon-photon repulsion and the reduction of the cavity photon fluctuations, the normal

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mode splitting weakens and the effective temperature of the moving mirror increases[19].

The above mentioned interesting results motivated us towards investigation of an optomechanical system which contains a nonlinear crystal consisting of a Kerr medium and a degenerate OPA. We will show that in the system under consideration the competition between the increasing of intracavity photon number due to the parametric gain process and the reduction of the number of photons due to the photon blockade mechanism arising from the presence of the Kerr medium leads to some new interesting effects in the dynamics of the movable mirror in the low photon number limit and the resolved sideband regime.

The paper is structured as follows. In Sec.II we describe the model, derive the quantum Langevin equations of motion for the system operators and find their steady-state mean values. In Sec.III we linearize the quantum Langevin equations of motion around the steady-state mean values, and analyze the stability conditions of the system. In Sec.IV we calculate the spectrum of small fluctuations in the position of the oscillating mirror, the effective frequency and effective damping rate of the mechanical oscillator, and analyze the influence of the nonlinear gain and anharmonicity parameter on them. In Sec.V we investigate the influences of the parametric gain and Kerr nonlinearity on the intensity and quadrature squeezing of the transmitted field. In Sec.VI we examine the entanglement between the optical and the mechanical mode. Finally, we summarize our conclusions in Sec.VII.

II. THE PHYSICAL MODEL

We consider a Kerr-down conversion optomechanical system composed of a degenerate OPA and a nonlinear Kerr medium placed within a Fabry-Perot cavity formed by a fixed partially transmitting mirror and one movable perfectly reflecting mirror in equilibrium with a thermal bath at temperature T . The movable mirror is free to move along the cavity axis and is treated as a quantum mechanical harmonic oscillator with effective mass m , frequency ω_m and energy decay rate $\gamma_m = \omega_m/Q$ (where Q is the mechanical quality factor). The cavity field is coherently driven by an input laser field with frequency ω_L and amplitude ϵ through the fixed mirror. Furthermore, the system is pumped by a coupling field to produce parametric oscillation and induce the Kerr nonlinearity in the cavity. In our investigation, we restrict the model to the case of single-cavity and mechanical modes[20, 21]. The single cavity-mode assumption is justified in the adiabatic limit, i.e., $\omega_m \ll \pi c/L$ which c is the speed of light in vacuum and L is the cavity length in the absence of the cavity field. We also assume that the induced resonance frequency shift of the cavity and the nonlinear parameter of the Kerr medium are much smaller than the longitudinal-mode spacing in the cavity. Furthermore, one can restrict to a single mechanical mode when

the detection bandwidth is chosen such that it includes only a single, isolated, mechanical resonance and mode-mode coupling is negligible. It should be noted that in the adiabatic limit, the number of photons generated by the Casimir, retardation, and Doppler effects is negligible [22–24].

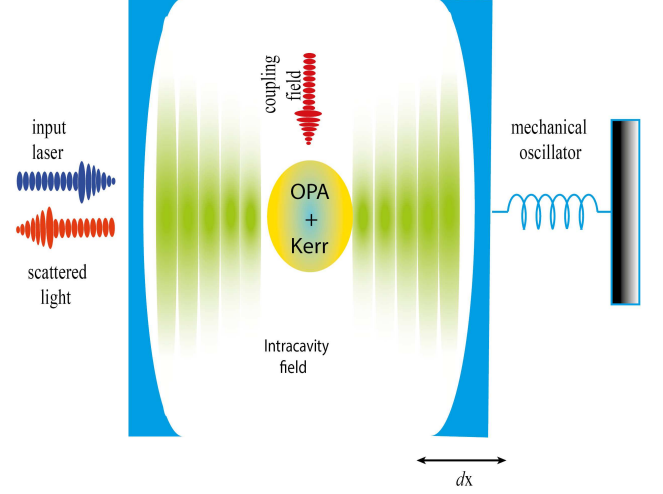


FIG. 1. (Color online) Schematic picture of the setup studied in the text. The cavity contains a Kerr-down conversion system which is pumped by a coupling field to produce parametric oscillation and induce Kerr nonlinearity in the cavity.

Under these conditions, the total Hamiltonian of the system in a frame rotating at the laser frequency ω_L can be written as

$$H = H_0 + H_1 \quad (1)$$

where,

$$H_0 = \hbar(\omega_0 - \omega_L)a^\dagger a + \left(\frac{p^2}{2m} + \frac{1}{2}m\omega_m^2 q^2\right) - \hbar g_m a^\dagger a q + i\hbar\epsilon(a^\dagger - a), \quad (2a)$$

$$H_1 = i\hbar G(e^{i\theta}a^{\dagger 2} - e^{-i\theta}a^2) + \hbar\eta a^{\dagger 2}a^2. \quad (2b)$$

The first two terms in H_0 are, respectively, the free Hamiltonian of the cavity field with annihilation(creation) operator $a(a^\dagger)$ and the movable mirror with resonance frequency ω_m and effective mass m , the third term describes the optomechanical coupling between the cavity field and the mechanical oscillator due to radiation pressure force, and the last term in H_0 describes the driving of the intracavity mode with the input laser. Also, the two terms in H_1 describes, respectively, the coupling of the intracavity field with the OPA and the Kerr medium; G is the nonlinear gain of the OPA which is proportional to the pump power driving amplitude, θ is the phase of the field driving the OPA, and η is the anharmonicity parameter proportional to the third order nonlinear susceptibility $\chi^{(3)}$ of the Kerr medium. The input laser field populates

the intracavity mode through the partially transmitting mirror, then the photons in the cavity will exert a radiation pressure force on the movable mirror. In a realistic treatment of the dynamics of the system, the cavity-field damping due to the photon-leakage through the incomplete mirror and the Brownian noise associated with the coupling of the oscillating mirror to its thermal bath should be considered. Using the input-output formalism of quantum optics[25], we can consider the effects of these sources of noise and dissipation in the quantum Langevin equations of motion. For the given Hamiltonian (1), we obtain the following nonlinear equations of motion

$$\dot{q} = \frac{p}{m}, \quad (3a)$$

$$\dot{p} = -m\omega_m^2 q + \hbar g_m a^\dagger a - \gamma_m p + \xi, \quad (3b)$$

$$\dot{a} = -i(\omega_0 - \omega_L)a + i g_m q a + \epsilon - 2i\eta a^\dagger a^2 + 2G a^\dagger e^{i\theta} - \kappa a + \sqrt{2\kappa} a_{in}, \quad (3c)$$

where κ is the cavity decay rate through the input mirror and a_{in} is the input vacuum noise operator that is characterized by the following correlation functions [25]

$$\langle \delta a_{in}(t) \delta a_{in}^\dagger(t') \rangle = \delta(t - t'), \quad (4a)$$

$$\langle \delta a_{in}(t) \delta a_{in}(t') \rangle = \langle \delta a_{in}^\dagger(t) \delta a_{in}^\dagger(t') \rangle = 0. \quad (4b)$$

The Brownian noise operator ξ describes the heating of the mirror by the thermal bath at temperature T and is characterized by the following correlation function [24]

$$\langle \xi(t) \xi(t') \rangle = \frac{\hbar \gamma_m m}{2\pi} \int \omega e^{-i\omega(t-t')} [\coth(\frac{\hbar\omega}{2k_B T}) + 1] d\omega, \quad (5)$$

where k_B is the Boltzmann constant.

We are interested in the steady-state regime and in small fluctuations with respect to the steady state. Thus we obtain the steady-state mean values of p , q and a as

$$p_s = 0, q_s = \frac{\hbar g_m}{m\omega_m^2} |a_s|^2, \quad (6)$$

$$a_s = \frac{\kappa - i\Delta' + 2Ge^{i\theta}}{\Delta'^2 + \kappa^2 - 4G^2} \epsilon, \quad (7)$$

where q_s denotes the new equilibrium position of the movable mirror and $\Delta' = \omega_0 - \omega_L - g_m q_s + 2\eta |a_s|^2 = \Delta + 2\eta |a_s|^2$ is the effective detuning of the cavity which includes both the radiation pressure and the Kerr medium effects. It is obvious that the optical path and hence the cavity detuning are modified in an intensity-dependent way. The first modification which is a mechanical nonlinearity, arises from the radiation pressure-induced coupling between the movable mirror and the cavity field and the second modification comes from the presence of the nonlinear Kerr medium in the optomechanical system. It has been shown that [26] for a given sufficiently large pure $\chi^{(3)}$ nonlinearity inside an optical cavity, the effect of photon blockade occurs as a consequence of large phase

shifts of the cavity detuning. It has also been predicted [27] that the same photon-photon interaction can occur in the strong coupling regime in the optomechanical systems. In our treatment the photon-photon interaction due to the radiation pressure is ignorable in comparison with photon-photon interaction due to the Kerr nonlinearity. Since Δ' satisfies a fifth-order equation, it can have five real solutions and hence the system may exhibit multistability for a certain range of parameters.

III. DYNAMICS OF SMALL FLUCTUATIONS

In order to investigate the dynamics of the system, we need to study the dynamics of small fluctuations near the steady state. We assume that the nonlinearity in the system is weak and decompose each operator in Eq.(3) as the sum of its steady-state value and a small fluctuation with zero mean value,

$$a = a_s + \delta a, \quad q = q_s + \delta q, \quad p = p_s + \delta p. \quad (8)$$

Inserting the above linearized forms of the system operators into Eq.(3), the linearized quantum Langevin equations for the fluctuation operators take the form

$$\frac{d}{dt} \delta q = \delta p / m, \quad (9a)$$

$$\frac{d}{dt} \delta p = -m\omega_m^2 \delta q + \hbar g_m (a_s \delta a^\dagger + a_s^* \delta a) - \gamma_m \delta p + \xi \quad (9b)$$

$$\frac{d}{dt} \delta a = -(i\Delta + \kappa) \delta a - i g_m a_s^* \delta q + (2G e^{i\theta} - 2i\eta a_s^2) \delta a^\dagger - 4i\eta |a_s|^2 \delta a + \sqrt{2\kappa} \delta a_{in}, \quad (9c)$$

$$\frac{d}{dt} \delta a^\dagger = (i\Delta - \kappa) \delta a^\dagger + i g_m a_s \delta q + (2G e^{-i\theta} + 2i\eta a_s^{*2}) \delta a + 4i\eta |a_s|^2 \delta a^\dagger + \sqrt{2\kappa} \delta a_{in}^\dagger. \quad (9d)$$

Defining the cavity field quadratures $\delta x = \delta a + \delta a^\dagger$ and $\delta y = i(\delta a^\dagger - \delta a)$ and the input noise quadratures $\delta x_{in} = \delta a_{in} + \delta a_{in}^\dagger$ and $\delta y_{in} = i(\delta a_{in}^\dagger - \delta a_{in})$ Eq.(9) can be written in the compact matrix form

$$\dot{u} = M u(t) + n(t), \quad (10)$$

where $u(t) = (\delta q, \delta p, \delta x, \delta y)^T$ is the vector of fluctuations, $n(t) = (0, \xi, \sqrt{2\kappa} \delta x_{in}, \sqrt{2\kappa} \delta y_{in})^T$ is the vector of noise sources and the matrix M is given by

$$M = \begin{pmatrix} 0 & 1/m & 0 & 0 \\ -m\omega_m^2 & -\gamma_m & \hbar a_+ & -i\hbar a_- \\ 2ia_- & 0 & -\kappa + \Gamma_1 & \Delta_1 + \delta_1 \\ 2a_+ & 0 & -\Delta_1 + \delta_1 & -\kappa - \Gamma_1 \end{pmatrix}, \quad (11)$$

where we have defined

$$a_\pm = g_m (a_s \pm a_s^*) / 2, \quad (12)$$

$$\Gamma_1 = 2G \cos \theta - i\eta (a_s^2 - a_s^{*2}),$$

$$\Delta_1 = \Delta + 4\eta |a_s|^2$$

$$\delta_1 = 2G \sin \theta - \eta (a_s^2 + a_s^{*2}). \quad (13)$$

Here, we concentrate on the stationary properties of the system. For this purpose, we should consider the steady-state condition governed by Eq.(10). The system is stable only if the real part of all eigenvalues of the matrix M are negative, which is also the requirement of the validity of the linearized method. The parameter region in which the system is stable can be obtained from the Routh-Hurwitz criterion[28], which gives the following three independent conditions:

$$s_1 = 2\kappa R_1 + \omega_m^2(\gamma_m + 4\kappa^2) > 0, \quad (14a)$$

$$s_2 = \{R_1\omega_m^2 - 4\frac{\hbar g_m^2}{m}\eta|a_s|^4 + iG(a_s^2 e^{-i\theta} - c.c.)\} > 0, \quad (14b)$$

$$s_3 = -[R_1\omega_m^2 - R_2] \times \omega_m^2(\gamma_m + 2\kappa) + [2\kappa R_1 + \omega_m^2(\gamma_m + 4\kappa^2)] > 0, \quad (14c)$$

where we have defined R_1 and R_2 as

$$R_1 = \Delta_1^2 - 4G^2 + \kappa^2 + 12\eta^2|a_s|^4 + 8\eta\Delta_1|a_s|^2 + 4iG\eta(a_s^2 e^{-i\theta} - c.c.), \quad (15a)$$

$$R_2 = 2\frac{\hbar g_m^2}{m}(2\eta|a_s|^4 + iG(a_s^2 e^{-i\theta} - c.c.)). \quad (15b)$$

IV. DISPLACEMENT SPECTRUM OF THE OSCILLATING MIRROR

Since we are interested in the spectrum of fluctuations in position of the oscillating mirror in the presence of Kerr-down conversion nonlinearity, it is convenient to solve the time-domain equation of motion (10) by Fourier transforming it into the frequency domain. The Fourier transform of the time-domain operator $u(t)$ is

$$\tilde{u}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{-i\omega t} u(t). \quad (16)$$

Then solving the linearized quantum Langevin equation for the position fluctuations of the oscillating mirror yields

$$\delta q(\omega) = F_1(\omega)\xi(\omega) + F_2(\omega)\delta a_{in}(\omega) + F_3(\omega)\delta a_{in}^\dagger(\omega), \quad (17)$$

in which

$$\begin{aligned} F_1(\omega) &= \frac{1}{md(\omega)} \{(-\kappa - i\omega)^2 + \beta_2\}, \\ F_2(\omega) &= \frac{\sqrt{2\kappa}\hbar g_m}{md(\omega)} [-a_s^*(-\kappa + i\Delta' - i\omega) + 2Ge^{-i\theta}a_s], \\ F_3(\omega) &= F_2(-\omega)^*, \end{aligned} \quad (18)$$

with

$$d(\omega) = (\omega_m^2 - \omega^2 + i\omega\gamma_m)(\beta_2 + (\kappa + i\omega)^2) - \beta_1, \quad (19)$$

$$\begin{aligned} \beta_1 &= \frac{2\hbar g_m^2}{m}[\Delta'|a_s|^2 + iG(a_s^2 e^{-i\theta} - c.c.)], \\ \beta_2 &= -4G^2 + \Delta_1^2 - 4\eta^2|a_s|^4 + 4iG\eta(a_s^2 e^{-i\theta} - c.c.). \end{aligned} \quad (20)$$

The first term in Eq.(17) originates from the thermal noise while the second and third terms arise from the radiation pressure force. The displacement spectrum of the mirror is defined by

$$S_q(\omega) = \frac{1}{4\pi} \int d\Omega e^{-i(\omega+\Omega)t} \langle \delta q(\omega)\delta q(\Omega) + \delta q(\Omega)\delta q(\omega) \rangle. \quad (21)$$

Using the following correlation functions in the frequency domain

$$\langle \delta a_{in}(\omega)\delta a_{in}^\dagger(\Omega) \rangle = 2\pi\delta(\omega + \Omega), \quad (22)$$

$$\langle \xi(\omega)\xi(\Omega) \rangle = 2\pi\hbar\gamma_m\omega[1 + \coth(\frac{\hbar\omega}{2k_B T})]\delta(\omega + \Omega), \quad (23)$$

the displacement spectrum of the oscillating mirror is obtained as

$$\begin{aligned} S_q(\omega) &= \hbar|\chi|^2 \{m\gamma_m\omega \coth(\frac{\hbar\omega}{2k_B T}) + 2\frac{(\hbar g_m)^2\kappa}{m^2|d(\omega)|^2} \\ &\times \frac{|a_s|^2\delta_2^2 + 4GRe(a_s^2 e^{-i\theta}(\kappa + i\Delta'))}{(\beta_2 + \kappa^2 - \omega^2)^2 + 4\kappa^2\omega^2}\}, \end{aligned} \quad (24)$$

where $\delta_2^2 = (\kappa^2 + \omega^2 + 4G^2 + \Delta'^2)$.

A. Normal Mode Splitting

In order to determine the structure of the displacement spectrum of the moving mirror we need to determine the eigenvalues of the matrix iM as the solution of Eq.(10) in the frequency domain, or the zeros of $d(\omega)$ in the denominator of coefficients in Eq.(18). First we assume that the radiation pressure coupling g_m is zero and find the eigenvalues of iM as

$$-\frac{i\gamma_m}{2} \pm \sqrt{\omega_m^2 - \frac{\gamma_m^2}{4}}, \quad -i\kappa \pm \sqrt{\beta_2}. \quad (25)$$

Thus in the absence of optomechanical coupling the effective frequency of the normal modes depends on β_2 which is a function of nonlinear coefficients η , G and intracavity intensity. Now we assume that the optomechanical coupling g_m is not zero, but the decay rates γ_m and κ are negligibly small in comparison with ω_m and β_2 . Then the zeros of $d(\omega)$ are approximately given by

$$\omega_{\pm}^2 \cong \frac{1}{2}(\omega_m^2 + \beta_2 \pm \sqrt{(\omega_m^2 - \beta_2)^2 + 4\beta_1}). \quad (26)$$

Hence, the Kerr-down conversion medium can alter the radiation pressure contribution to the NMS in two ways. Firstly, both nonlinearity parameters G and η change the mean intracavity photon number considerably. Secondly the parameter β_1 is a function of the gain and anharmonicity parameters.

In Fig.2, by using the experimental parameters from Ref.[30], we have plotted the mean intracavity photon

number a_s and the normalized parameter β_1/β_0 versus the normalized gain G/κ for two different values of anharmonicity parameter η , where $\beta_0 = 2\hbar g_m^2(\Delta|a_s|^2/m)$ is the contribution of radiation pressure to the normal modes of displacement spectrum in the bare cavity. The figure shows that while there is a reduction in the number of intracavity photons (hence the radiation pressure) due to the photon-photon repulsion mechanism (in the presence of Kerr medium), the contribution of radiation pressure coupling to the normal modes which is contained in β_1 can be increased by choosing proper values of initial detuning and nonlinear coefficients G , η and θ in such a way that $\beta_1 > \beta_0$.

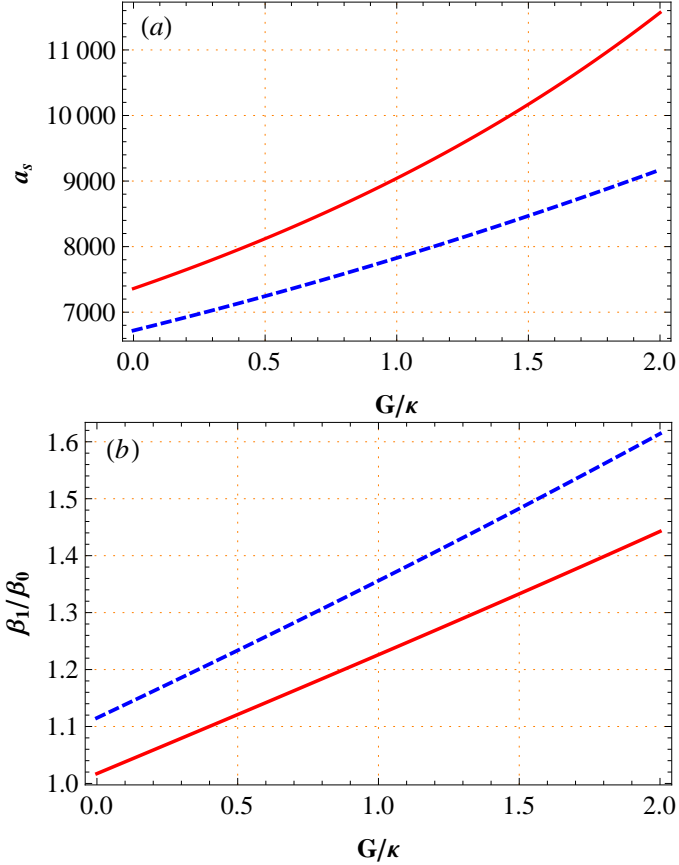


FIG. 2. (Color online)(a)The mean intracavity photon number and (b) the normalized parameter β_1/β_0 versus the normalized gain G/κ for $\eta = 0.01$ Hz (red solid line) and $\eta = 0.05$ Hz (blue dashed line).The parameters are $\omega_m/2\pi = 10$ MHz, $L = 1$ mm, $m = 10$ ng, $Q = 5 \times 10^5$ Hz, input laser power $P = 6.9$ mW at $\lambda = 1064$ nm, $\kappa = 0.1\omega_m$, and $\theta = \pi/2$.

It is well known that the NMS in a bare-damped optomechanical system occurs only if $g_0|a_s| \gtrsim \kappa$ (where $g_0 = \sqrt{\hbar/2m\omega_m g_m}$) due to the finite width of the peaks[15, 16]. In Fig.(3) we have plotted the displacement spectrum of the movable mirror below this threshold ($g_0|a_s| \gtrsim \kappa$). As expected, the radiation pressure-induced coupling between the oscillating mirror and the fluctuations of the cavity mode does not lead to NMS.

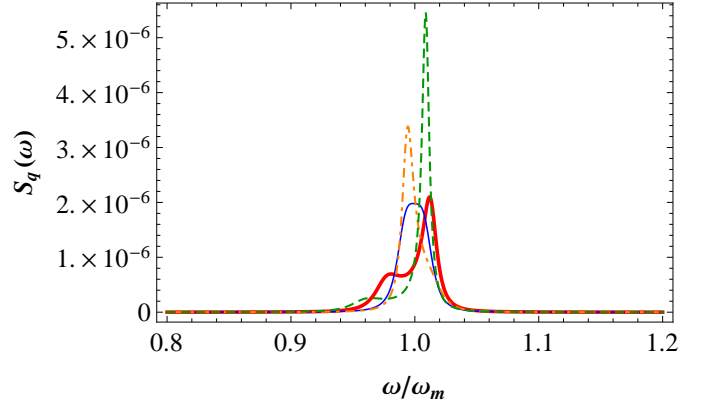


FIG. 3. (Color online)The displacement spectrum versus the normalized frequency ω/ω_m for bare cavity ($\eta = 0, G = 0$)(blue thin curve), for cavity with Kerr nonlinearity ($\eta = 0.01$ Hz, $G = 0$) (orange dashed-dotted curve), with gain nonlinearity ($\eta = 0, G = 8 \times 10^6$ Hz) (green dashed curve) and with both nonlinearity($\eta = 0.01$ Hz, $G = 8 \times 10^6$ Hz) (red thick curve). The parameters are $\omega_m/2\pi = 10$ MHz, $L = 3$ mm, $m = 12$ ng, $Q = 5 \times 10^5$ Hz, input laser power $P = 9$ mW at $\lambda = 1064$ nm, $\kappa = 0.02\omega_m$, and $\theta = \pi/2$.

The presence of the Kerr medium alone does not change this situation, while in the presence of the gain medium solely, mode splitting occurs only when the parametric gain G is large enough. However, as shown in Fig.(3), the combination of these two nonlinearities leads to the appearance of NMS in the displacement spectrum. We can come to this conclusion from the fact that in the presence of the two nonlinearities, there is a competition between the two quantum processes; the nonlinear gain medium increases the intracavity photon number and splits up the pump photons into two degenerate subharmonic photons, while the Kerr medium shifts the energy levels of the cavity proportional to the anharmonicity parameter η which blocks the entrance of a large number of photons into the cavity. The combination of the energy shift resulting from the Kerr medium and increasing the photon number due to the gain medium leads to the appearance of NMS. It is notable that the presence of the gain medium decreases the degree of photon-photon repulsion due to the Kerr nonlinearity, and hence increases the parameter β_1 in Eq.(26) and the radiation pressure effect on the oscillating mirror. In Fig.(4) and Fig.(5) we have plotted the displacement spectrum of the movable mirror above the threshold of splitting ($g_0|a_s| \gtrsim \kappa$) for different values of the gain coefficient G and different values of anharmonicity parameter η , respectively. As expected, the presence of the gain medium inside the cavity strengthens the coupling between the oscillating mirror and the cavity mode, while the presence of the Kerr medium inside the cavity weakens this coupling due to the photon-photon repulsion and shifts the modes far from each other.

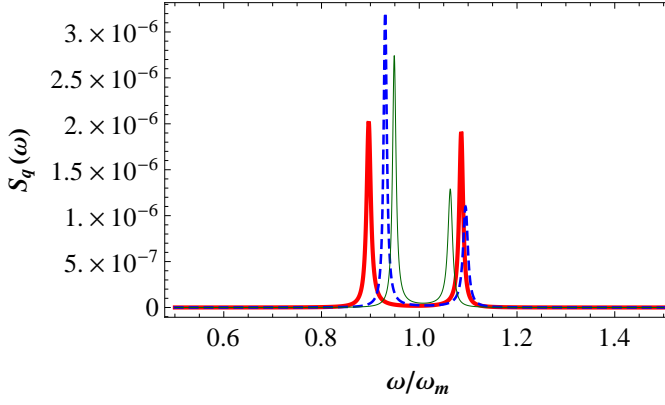


FIG. 4. (Color online) The displacement spectrum versus the normalized frequency ω/ω_m for different values of the parametric gain: $G = 0$ (green thin curve), $G = 10^5$ Hz (blue dashed curve), and $G = 10^7$ Hz (red thick curve). The parameters are $\eta = 0.01$ Hz, $\Delta = \omega_m$, $\theta = \pi/2$, $\omega_m/2\pi = 10$ MHz, $L = 1$ mm, effective mass $m = 10$ ng, $Q = 5 \times 10^5$ Hz, $P = 30$ mW, $\lambda = 1064$ nm, and $\kappa = 0.01\omega_m$.

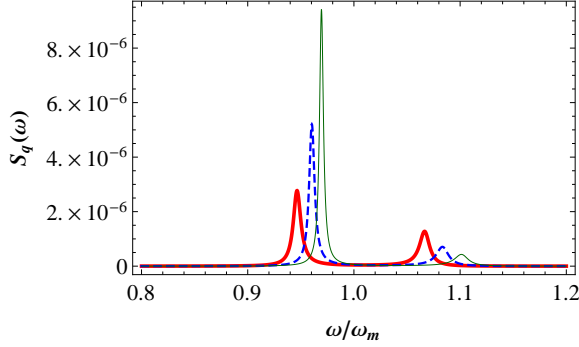


FIG. 5. (Color online) The displacement spectrum versus the normalized frequency ω/ω_m for different values of the anharmonicity: $\eta = 0.01$ Hz (red thick curve), $\eta = 0.03$ Hz (blue dashed curve), and $\eta = 0.06$ Hz (green thin curve). The parameters are $G = 10^6$ Hz, $\Delta = \omega_m$, $\theta = \pi/2$, $\omega_m/2\pi = 10$ MHz, $L = 1$ mm, $m = 10$ ng, $Q = 5 \times 10^5$ Hz, $P = 30$ mW, $\lambda = 1064$ nm, and $\kappa = 0.01\omega_m$.

B. Effective frequency and effective damping rate of the oscillating mirror

The radiation-pressure force can modify the dynamics of the mechanical oscillator and therefore the susceptibility of the oscillating mirror can be considered as the susceptibility of an oscillator, with an effective frequency and an effective damping. These quantities contain important information about the quantum behavior of the system. The radiation pressure-induced change in the frequency of the mechanical mode is known as the optical spring effect [32], and the radiation pressure-induced change in the damping rate of the mirror shows how the optical field acts effectively as a viscous fluid that damps the mirror oscillation and cools its center-of mass motion [33, 34]. In this section we investigate the effects

of the Kerr and the gain nonlinearities on the effective frequency ω_{eff} and the effective damping rate γ_{eff} of the oscillating mirror. We can find the mechanical susceptibility of the mirror according to the dependence of the $\delta q(\omega)$ on the fluctuations in the total force on the mirror $\delta F_T(\omega)$ [35],

$$\delta q(\omega) = \chi(\omega) \delta F_T(\omega). \quad (27)$$

According to Eq. (17), $\delta F_T(\omega)$ consists of a radiation pressure and a Brownian motion term. Thus the mechanical susceptibility of the mirror (χ), its effective resonance frequency (ω_{eff}), and its effective damping rate (γ_{eff}) are, respectively, given by

$$\chi^{-1} = m(\omega_{eff}^2 - \omega^2) + i\omega\gamma_{eff}, \quad (28)$$

$$\omega_{eff}^2 = \omega_m^2 - \frac{\beta_1(\beta_2 + \kappa^2 - \omega^2)}{|\beta_2 + (-\kappa + i\omega)^2|^2}, \quad (29)$$

$$\gamma_{eff} = m\left(\gamma_m + \frac{2\beta_1\kappa}{|\beta_2 + (-\kappa + i\omega)^2|^2}\right). \quad (30)$$

In Figs. 6(a) and 6(b) we have plotted the normalized effective frequency of the oscillating mirror ω_{eff}/ω_m versus the normalized frequency ω/ω_m for $\Delta = \omega_m$ and for different values of G [Fig. 6(a)] and different values of η [Fig. 6(b)]. As is seen, although the gain medium does not change the effective frequency of the moving mirror considerably, the energy shift of the cavity mode due to the Kerr nonlinearity manifests itself in the response frequency of the oscillating mirror. This means that the effective coupling between the cavity and the mechanical modes (and therefore the cooling of the mirror) takes place when the effective detuning of the cavity Δ' (but not Δ) is chosen near the oscillating mirror frequency. In Figs. 7(a) and 7(b) we have plotted the normalized effective damping rate of the oscillating mirror $\gamma_{eff}/m\gamma_m$ versus the normalized frequency ω/ω_m for $\Delta = \omega_m$ and for different values of G [Fig. 7(a)] and different values of η [Fig. 7(b)]. As is seen, by increasing the gain parameter the effective damping rate of the mirror motion increases while increasing the anharmonicity parameter not only leads to the frequency shift but also decreases the effective damping rate.

C. Effects of the Kerr-down conversion nonlinearity on the effective temperature of the mirror

Here we examine the effects of the Kerr and the gain nonlinearities on the back-action ground-state cooling of the oscillating mirror. In order to investigate the cooling of the mirror it is sufficient to consider the mean number of quanta of the vibrational excitation of the mirror as defined by [31]

$$n_m = \frac{k_B T_{eff}}{\hbar\omega_{eff}} = \frac{k_B T}{\hbar\omega_m} \frac{\gamma_m}{\gamma_{eff}} \left(\frac{\omega_m}{\omega_{eff}}\right)^3, \quad (31)$$

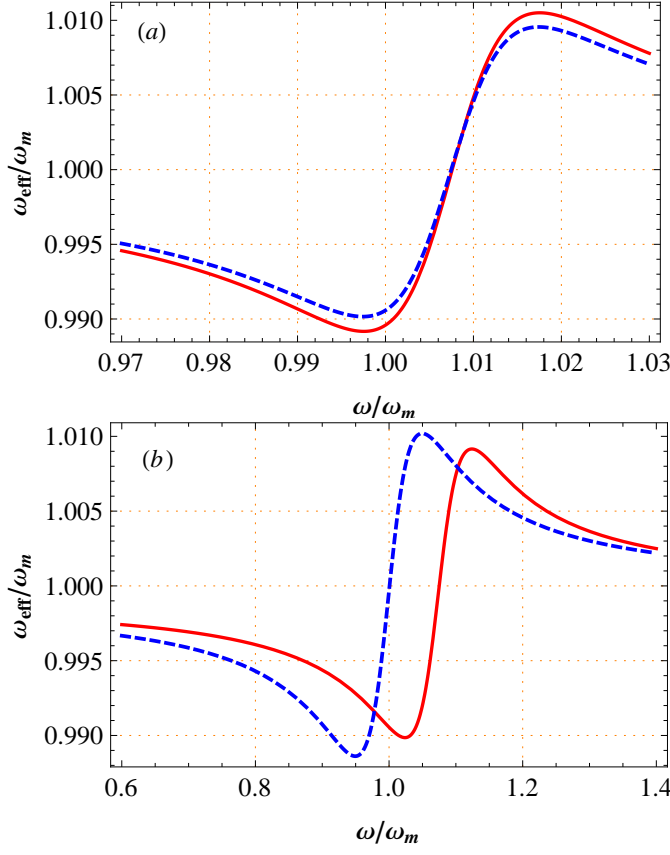


FIG. 6. (Color online) The normalized effective frequency ω_{eff}/ω_m versus the ω/ω_m (a) for $\eta = 0.01\text{Hz}$ and for different values of the gain parameter: $G = 0$ (blue dashed curve), and $G = 10^6\text{Hz}$ (red solid curve), (b) for $G = 10^6\text{Hz}$ and for different values of anharmonicity parameter: $\eta = 0$ (blue dashed curve), and $\eta = 0.01\text{Hz}$ (red solid curve). The parameters are $\omega_m/2\pi = 10\text{ MHz}$, $L = 2\text{mm}$, $m = 10\text{ ng}$, $Q = 5 \times 10^5\text{ Hz}$, $P = 6.9\text{ mW}$, $\kappa = 0.01\omega_m$, $\Delta = \omega_m$, and $\theta = \pi/2$.

where we expand the effective frequency and effective damping rate of the oscillating mirror in a Taylor series around $\omega = \omega_m$ and keep only the leading terms in the respective expansions

$$\begin{aligned}\omega_{eff}(\omega) &\sim \omega_{eff}(\omega_m) \equiv \omega_{eff}, \\ \gamma_{eff}(\omega) &\sim \gamma_{eff}(\omega_m) \equiv \gamma_{eff}.\end{aligned}\quad (32)$$

The ground-state cooling is approached if $n_m < 1$. In Figs.8(a) and 8(b) we have plotted, respectively, n_m and T_{eff} for the initial equilibrium temperature $T = 0.4\text{K}$ versus Δ/ω_m and for different values of G and η . It is evident that as G increases the system cools down to the ground-state ($n_m < 1$) while as η increases the effective temperature of the system increases. The heating of the system in the presence of the Kerr medium is due to the energy shift of the cavity which weakens the field-mirror coupling and reduces the number of intracavity photons because of the photon-photon repulsion mechanism.

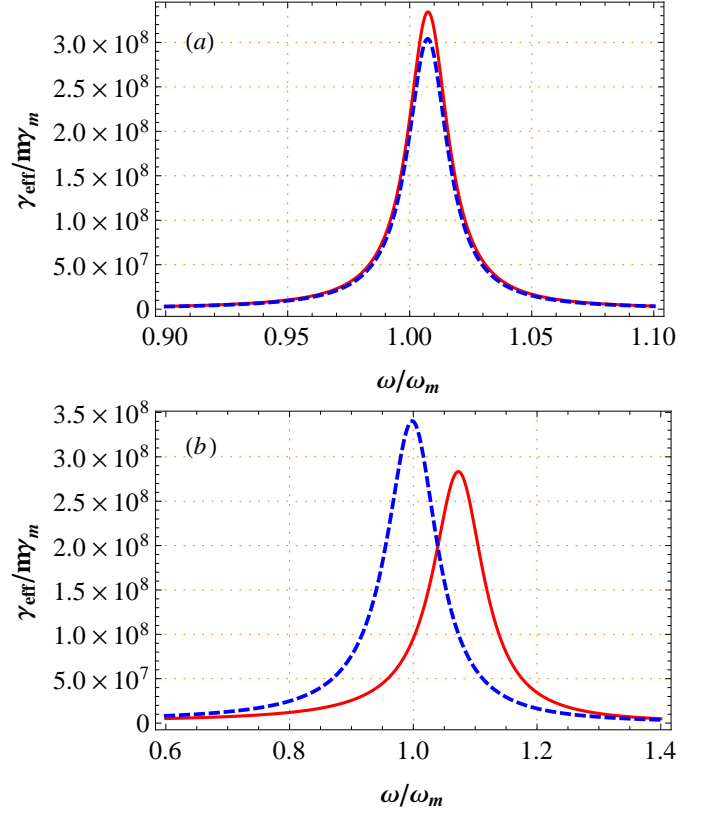


FIG. 7. (Color online) The normalized effective damping rate of the oscillating mirror $\gamma_{eff}/m\gamma_m$ versus the ω/ω_m (a) for $\eta = 0.01\text{Hz}$ and for different values of gain parameter: $G = 0$ (blue dashed curve), and $G = 10^6\text{Hz}$ (red solid curve), (b) for $G = 10^6\text{Hz}$ and for different values of anharmonicity parameter: $\eta = 0$ (blue dashed curve), and $\eta = 0.01\text{Hz}$ (red solid curve). The parameters are $\omega_m/2\pi = 10\text{ MHz}$, $L = 2\text{mm}$, $m = 10\text{ ng}$, $Q = 5 \times 10^5\text{ Hz}$, $P = 6.9\text{ mW}$, $\kappa = 0.01\omega_m$, $\Delta = \omega_m$, and $\theta = \pi/2$.

V. EFFECTS OF THE KERR-DOWN CONVERSION NONLINEARITY ON THE INTENSITY AND QUADRATURE NOISE SPECTRA OF THE TRANSMITTED FIELD

The intensity spectrum of the transmitted field is given by the Fourier transform of the two time correlation functions $\langle \delta a_{out}^\dagger(t + \tau) \delta a_{out}(t) \rangle$:

$$S(\omega) = \int_{-\infty}^{\infty} d\tau \langle \delta a_{out}^\dagger(t + \tau) \delta a_{out}(t) \rangle e^{-i\omega\tau}. \quad (33)$$

Solving the matrix equation (10) we obtain the solution for the cavity-field fluctuation operator δa and then using the input-output relation $a_{out} = \sqrt{2\kappa}a - a_{in}$, we obtain the fluctuations of the output field to be

$$\delta a_{out}(\omega) = V_1(\omega)\xi(\omega) + V_2(\omega)\delta a_{in}(\omega) + V_3(\omega)\delta a_{in}^\dagger(\omega), \quad (34)$$

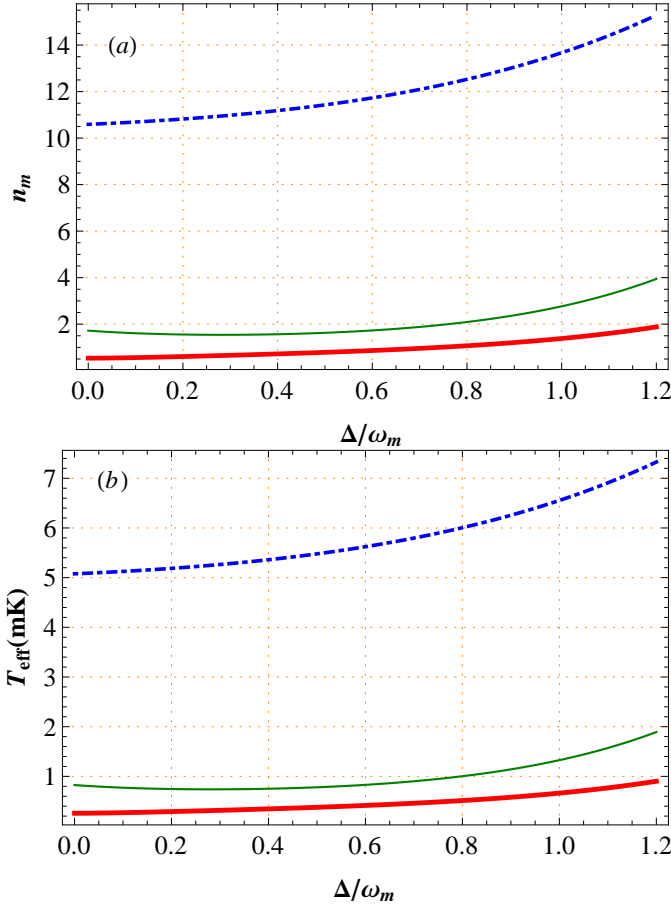


FIG. 8. (Color online) Plots of (a) the mean number of vibrational quanta n_m and (b) the effective temperature T_{eff} for $G = 2 \times 10^7 \text{ Hz}$, $\eta = 0.05 \text{ Hz}$ (blue dotted-dashed curve), $G = 2 \times 10^7 \text{ Hz}$, $\eta = 0.01 \text{ Hz}$ (red thick curve) and $G = 0$, $\eta = 0.01 \text{ Hz}$ (green thin curve). The parameters are $\omega_m/2\pi = 10 \text{ MHz}$, $L = 10 \text{ mm}$, $m = 10 \text{ ng}$, $Q = 5 \times 10^5 \text{ Hz}$, $P = 6.9 \text{ mW}$, $\kappa = 0.7\omega_m$, and $\theta = \pi/2$.

and $\delta a_{out}^\dagger(\omega) = [\delta a_{out}(-\omega)]^\dagger$, where

$$\begin{aligned} V_1(\omega) &= \sqrt{2\kappa} \frac{ig_m}{Md(\omega)} \{a_s(\kappa - i\Delta' - i\omega) - 2Ga_s^* e^{i\theta}\}, \\ V_2(\omega) &= \frac{2\kappa}{d(\omega)} \left\{ \frac{i\hbar g_m^2 a_s^2}{M} + (\omega_m^2 - \omega^2 + i\omega\gamma_m) \right. \\ &\quad \left. (i\delta_1 + \Gamma) \right\} - 1, \\ V_3(\omega) &= \frac{2\kappa}{d(\omega)} \left\{ \frac{i\hbar g_m^2 |a_s|^2}{M} + (\omega_m^2 - \omega^2 + i\omega\gamma_m) \right. \\ &\quad \left. (\kappa + i(\omega + \Delta_1)) \right\}, \end{aligned} \quad (35)$$

In Eq.(34) the first term stems from the mechanical oscillator thermal noise, while the other two terms are from the input vacuum noise. Making use of the correlation peroperties for the noise forces and Eqs.(34) and (35), we obtain the output field spectrum as

$$S(\omega) = |V_3(-\omega)|^2 + m\hbar\gamma_m\omega[1 + \coth(\frac{\hbar\omega}{2k_B T})]|V_1(-\omega)|^2. \quad (36)$$

The quadrature noise spectrum of the transmitted filed is given by[36]

$$\begin{aligned} S_\varphi(\omega) &= \int_{-\infty}^{\infty} d\tau \langle \delta x_\varphi^{out}(t + \tau) \delta x_\varphi^{out}(\tau) \rangle_{ss} e^{-i\omega\tau} \\ &= \langle \delta x_\varphi^{out}(\omega) \delta x_\varphi^{out}(\omega) \rangle, \end{aligned} \quad (37)$$

where $\delta x_\varphi^{out}(\omega) = e^{-i\varphi} \delta a_{out}(\omega) + e^{i\varphi} \delta a_{out}^\dagger(\omega)$ is the Fourier trnasform of the output quadrature, with φ as its externally controllable phase angle which is experimentally measurable in a homodyne detection scheme[37]. For the system under consideration quadrature noise spectrum is given by

$$\begin{aligned} S_\varphi(\omega) &= e^{-2i\varphi} C_{aa}^{out}(\omega) + e^{2i\varphi} C_{aa}^{out*}(\omega) \\ &\quad + C_{a^\dagger a}^{out}(\omega) + C_{aa^\dagger}^{out}(\omega), \end{aligned} \quad (38)$$

where

$$\begin{aligned} C_{aa}^{out} &= \{m\hbar\gamma_m\omega[1 + \coth(\frac{\hbar\omega}{2k_B T})]V_1(\omega)V_1(-\omega) \\ &\quad + V_2(\omega)V_3(-\omega)\}, \end{aligned} \quad (39)$$

$$\begin{aligned} C_{aa^\dagger}^{out} &= \{m\hbar\gamma_m\omega[1 + \coth(\frac{\hbar\omega}{2k_B T})]|V_1(\omega)|^2 \\ &\quad + |V_2(\omega)|^2\}, \end{aligned} \quad (40)$$

$$\begin{aligned} C_{a^\dagger a}^{out} &= \{m\hbar\gamma_m\omega[1 + \coth(\frac{\hbar\omega}{2k_B T})]|V_1(-\omega)|^2 \\ &\quad + |V_3(-\omega)|^2\}. \end{aligned} \quad (41)$$

We define the optimum quadrature squeezing $S_{opt}(\omega)$ by choosing $\varphi(\omega)$ in such a way that $dS_\varphi(\omega)/d\varphi = 0$. This yields

$$e^{2i\varphi_{opt}} = -\frac{C_{aa}^{out}}{|C_{aa}^{out}|}. \quad (42)$$

Then, substituting back into Eq.(38), we obtain the optimized squeezing spectrum as

$$S_{opt}(\omega) = -2|C_{aa}^{out}| + C_{a^\dagger a}^{out}(\omega) + C_{aa^\dagger}^{out}(\omega). \quad (43)$$

In Figs.9(a) and 9(b) we have plotted, respectively, the intensity and the optimized squeezing spectra of the transmitted field for different values of the gain nonlinearity versus the normalized response frequency ω/ω_m . As can be seen increasing the gain parameter G causes the intensity and the degree of squeezing of the transmitted field to increase. The shift in the intensity spectrum is due to the energy shift of the cavity in the presence of the Kerr medium. In Figs.10(a) and 10(b) we have plotted, respectively, the intensity and the optimized squeezing spectra of the transmitted field for different values of the anharmonicity parameter η versus the normalized response frequency ω/ω_m . It can be seen as η increases the intensity and the degree of squeezing of the transmitted field increases.

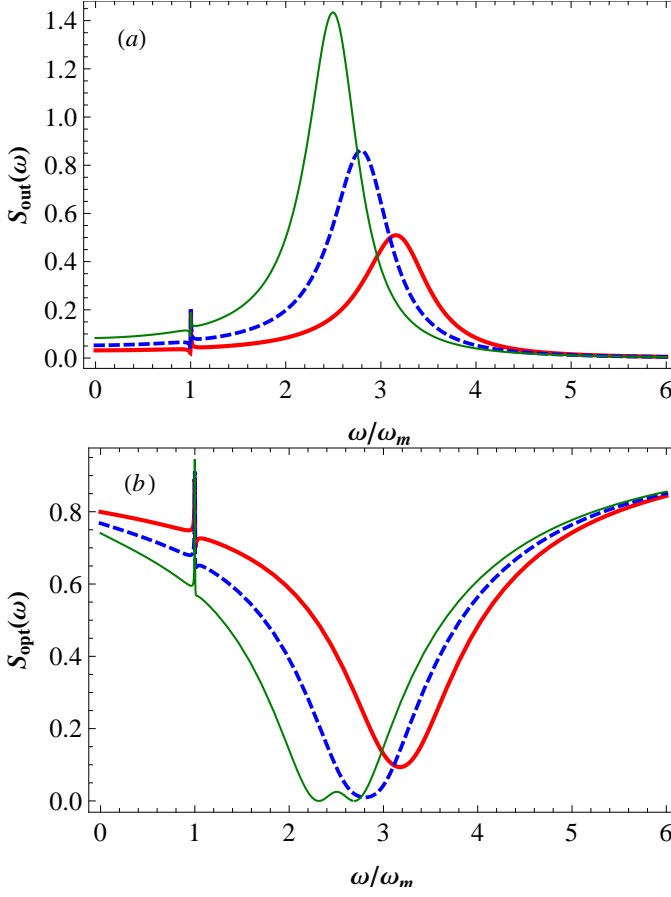


FIG. 9. (Color online)(a) The intensity spectrum and (b) the squeezing spectrum of the transmitted field versus the normalized frequency ω/ω_m for $G = 1 \times 10^7$ Hz (red thick curve), $G = 2 \times 10^7$ Hz (blue dashed curve), and $G = 3 \times 10^7$ Hz (green thin curve). The parameters are $\eta = 0.01$ Hz, $\omega_m/2\pi = 10$ MHz, $L = 10$ mm, $m = 10$ ng, $Q = 5 \times 10^5$ Hz, $P = 6.9$ mW, $\kappa = 0.7\omega_m$, and $\theta = \pi/2$.

VI. ENTANGLEMENT PROPERTIES OF THE SYSTEM

It has been shown previously [38] that within each optomechanical cavity the photon-phonon entanglement can be generated by means of radiation pressure. Here we are interested in the generation of stationary entanglement between the movable mirror and the cavity field in the presence of Kerr-down conversion medium. Since our system is in a Gaussian state and the linear nature of Eq.(10) preserves the Gaussian nature of the initial state of the system, we can use the logarithmic negativity measure [39] to quantify the entanglement. When the stability conditions of Eq.(14) are fulfilled, we can solve Eq.(10) for the 4×4 stationary correlation matrix (CM) V

$$MV + VM^T = -D, \quad (44)$$

where the elements of the correlation matrix V are defined as $V_{ij} = \langle u_i u_j + u_j u_i \rangle / 2$ and $D =$

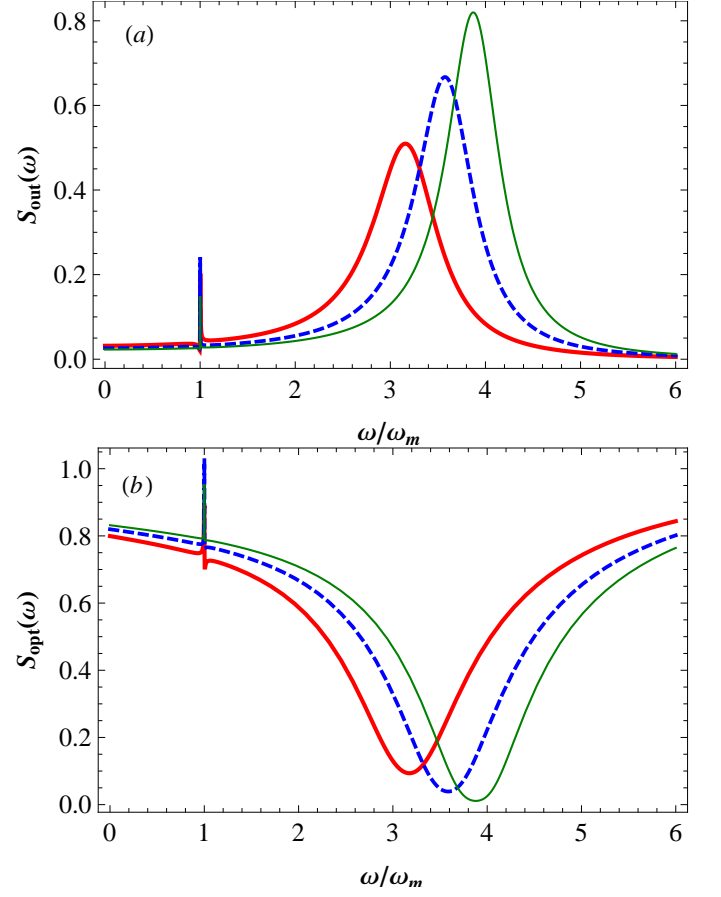


FIG. 10. (Color online)(a) The intensity spectrum and (b) the squeezing spectrum of the transmitted field versus the normalized frequency ω/ω_m for $\eta = 0.01$ Hz (red thick curve), $\eta = 0.03$ Hz (blue dashed curve), and $\eta = 0.05$ Hz (green thin curve). The parameters are $G = 10^7$ Hz, $\omega_m/2\pi = 10$ MHz, $L = 10$ mm, $m = 10$ ng, $Q = 5 \times 10^5$ Hz, $P = 6.9$ mW, $\kappa = 0.7\omega_m$, and $\theta = \pi/2$.

$\text{Diag}[0, m\hbar\gamma_m\omega_m(2\bar{n} + 1), \kappa, \kappa]$ is the diagonal diffusion matrix in which we used the following approximation

$$\omega \coth\left(\frac{\hbar\omega}{2k_B T}\right) \simeq \omega_m \frac{2k_B T}{\hbar\omega_m} \simeq \omega_m(2\bar{n} + 1), \quad (45)$$

where $\bar{n} = [e^{\hbar\omega_m/k_B T} - 1]^{-1}$. Then the photon-phonon entanglement can be quantified by the logarithmic negativity E_N as

$$E_N = \max[0, -\ln 2\eta^-], \quad (46)$$

where $\eta^- \equiv 2^{-1/2} [\Sigma(V) - \sqrt{\Sigma(V)^2 - 4\det V}]^{1/2}$, is the lowest symplectic eigenvalue of the partial transpose of the 4×4 CM, V , with $\Sigma(V) = \det V_A + \det V_B - 2\det V_C$, and we used the 2×2 block form of the CM

$$\begin{pmatrix} V_A & V_C \\ V_C^T & V_B \end{pmatrix}, \quad (47)$$

with V_A associated to the oscillating mirror, V_B to the cavity mode, and V_C describing the optomechanical cor-

relations. In Fig.(11) we have plotted E_N versus the nor-

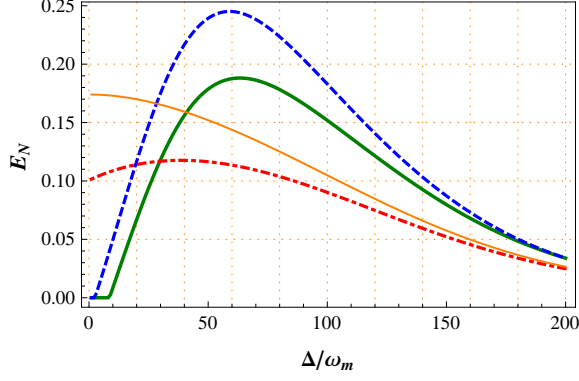


FIG. 11. (Color online) The logarithmic negativity E_N versus the normalized detuning Δ/ω_m for $(\eta = 0, G = 0)$ (green thick curve), $(\eta = 0.01\text{Hz}, G = 0)$ (red dashed-dotted curve), $(\eta = 0, G = 6 \times 10^6\text{Hz})$ (blue dashed curve) and $(\eta = 0.01\text{Hz}, G = 6 \times 10^6\text{Hz})$ (orange thin curve). The parameters are $\omega_m/2\pi = 10\text{ MHz}$, $L = 1\text{mm}$, $m = 10\text{ ng}$, $Q = 5 \times 10^5$ Hz, $P = 15\text{ mW}$, $\kappa = 0.9\omega_m$, and $\theta = \pi/2$.

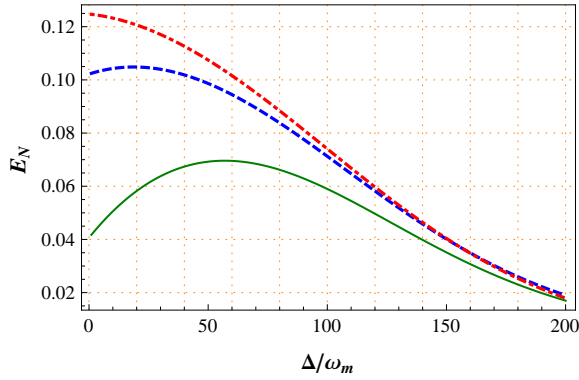


FIG. 12. (Color online) The logarithmic negativity E_N versus the normalized detuning Δ/ω_m for $G = 10^3\text{ Hz}$ (green solid curve), $G = 10^6\text{ Hz}$ (red dashed-dotted curve), and $G = 10^7\text{ Hz}$ (blue dashed curve). The parameters are $\eta = 0.01\text{ Hz}$, $\omega_m/2\pi = 10\text{ MHz}$, $L = 1\text{mm}$, $m = 10\text{ ng}$, $Q = 5 \times 10^5$ Hz, $P = 15\text{ mW}$, $\kappa = 0.9\omega_m$, and $\theta = \pi/2$.

malized detuning Δ/ω_m for a bare cavity, a cavity that contains only the Kerr medium, a cavity that contains only the gain medium and a cavity that contains both the Kerr and gain medium. We see that, as expected, while the Kerr medium reduces the degree of phonon-photon entanglement, the gain medium increases it considerably. This result arises from the fact that the radiation pressure, which is responsible for the phonon-photon entanglement, increases considerably in the presence of the gain nonlinearity while decreases in the presence of the Kerr nonlinearity. Therefore similarly to what happens for the mirror cooling process there is a competition

between the gain and Kerr nonlinearities for increasing and decreasing quantum effects, respectively. In Fig.12 we have plotted E_N versus the normalized detuning for different values of the gain nonlinearity. It can be seen that as G increases the degree of entanglement increases. It should be noted that we cannot increase G arbitrarily because of the stability condition of the system.

VII. SUMMARY AND CONCLUSIONS

In this paper we considered the canonical optomechanical cavity composed of a partially transmitting mirror and one perfectly reflecting movable mirror containing a Kerr-down conversion nonlinear medium. For this system we investigated the influence of the two nonlinearities on the dynamics of the oscillating mirror, the intensity and squeezing spectra of the transmitted field and the steady-state mirror-field entanglement. For the dynamics of the oscillating mirror we have found that in the peresence of the two nonlinearities, there is a competition between the two quantum processes; the splitting of the pump photons in two degenerate subharmonic photons as well as increasing the intracavity photon number due to the nonlinear gain medium, and shifting the energy level of the cavity proportional to the anharmonicity parameter η which blocks the entrance of a large number of photons into the cavity. This competition can lead to enhancement of the radiation pressure contribution to the normal modes of the cavity and NMS below the threshold $g_0 a_s \lesssim \kappa$. Also, it was demonstrated that in the presence of the Kerr-down conversion nonlinearity the ground-state cooling of the mechanical mirror is possible. Although with increasing the anharmonicity parameter η the vibrational excitation and effective temperature of the mirror increases, the enhancement of the gain medium can compensate the increasing of the phonon number to some extent. In the investigation of the intensity spectrum and quadrature squeezing of the transmitted field, it was shown that the range of response frequency of the field undergoes a shift because of anharmonicity parameter η but in this range of the frequency both the Kerr and gain media increase the degree of squeezing. Finally, we investigated the influence of the Kerr-down conversion on the degree of stationary entanglement between the cavity and the mechanical modes. It was shown that the Kerr nonlinearity reduces the degree of entanglement considerably, while the gain medium increases it.

ACKNOWLEDGEMENT

The authors wish to thank The Office of Graduate Studies of The University of Isfahan for their support.

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